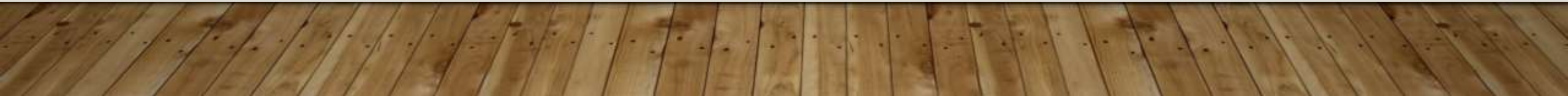


RINGS



RINGS

DEFINITION:

Let R be a non empty set and $(+, \cdot)$ be two binary operations on R Then the Algebraic structure $(R, +, \cdot)$ is said to be Ring if

a) $(R, +)$ is a commutative group

1 ASSOCIATIVE PROPERTY

$$. a+(b+c)=(a+b)+c \quad \forall a,b,c \in R$$

2 IDENTITY PROPERTY

For every $a \in R \exists 0 \in R \ni a+0 = a=0+a$. Where 0 is identity element in R

3 INVERSE PROPERTY

For each $a \in R \exists -a \in R \exists a + (-a) = 0 = (-a) + a$. Where $-a$ is inverse of a in R .

4 COMMUTATIVE PROPERTY

$a + b = b + a \forall a, b \in R$.

b) (R, \cdot) is a semi group

. $a \cdot (b \cdot c) = (a \cdot b) \cdot c \forall a, b, c \in R$

c) Distributive law's holds in R

1 $a \cdot (b + c) = a \cdot b + a \cdot c$ (LDL) $\forall a, b, c \in R$

2 $(b + c) \cdot a = b \cdot a + c \cdot a$ (RDL) $\forall a, b, c \in R$

EXAMPLE:

1 $(\mathbb{N}, +, \cdot)$ is not a Ring because the identity element '0' does not exist in \mathbb{N}

2 $(\mathbb{W}, +, \cdot)$ is not a Ring because the inverse does not exist in \mathbb{W}

3 $(\mathbb{Z}, +, \cdot)$ is a Ring

4 $(\mathbb{Q}, +, \cdot)$ is a Ring

TYPES OF RINGS

RING WITH UNITY

Let $(R, +, \cdot)$ be a ring, if R satisfies for $a \in R \exists 1 \in R \ni a \cdot 1 = a = 1 \cdot a$ then R is said to be Ring with unity.

COMMUTATIVE RING

Let $(R, +, \cdot)$ be a ring, if R satisfies

$a \cdot b = b \cdot a$ for $a, b \in R$ then R is called commutative Ring.

BOOLEAN RING

Let R be a Ring. If $a^2 = a \forall a \in R$ then R is called BOOLEAN Ring.

THEOREM

Statement : Let R be a Ring $0, a, b \in R$ then

- I. $0a = a0 = 0$
- II. $a(-b) = (-a)b = -ab$
- III. $(-a)(-b) = ab$
- IV. $a(b-c) = ab - ac$

PROOF:

Let R be a Ring $0, a, b \in R$

CLAIM:

Case I $0a=a0=0$

Consider $0a=(0+0)a$

$0a=0a+0a$

$0=0a$

Similarly $0=a0$

$a0=0a=0$

Case ii $a(-b) = (-a)b = -ab$

Consider $a(-b) + ab$

$$= a(-b + b)$$

$$= a0$$

$$= 0 \text{ (By case I)}$$

Now $a(-b) + ab = 0$

$$a(-b) = -(ab)$$

Similarly $(-a)b = -(ab)$

Case iii $(-a)(-b)=ab$

Consider $(-a)(-b)$

$$= -[a(-b)]$$

$$= -[-(ab)] \quad (\text{case ii})$$

$$= ab$$

$$(-a)(-b) = ab$$

Case iv. $a(b-c)=ab-ac$

Consider $a(b-c)$

$$= a(b+(-c))$$

$$= ab+a(-c)$$

$$= ab-ac$$

Therefore $a(b-c)=ab-ac$