RINGS

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DEFINITION:

Let R be a non empty set and (+,•) be two binary operations on R Then the Algebraic structure (R,+,•) is said to be Ring if

a) (R,+) is a commutative group

I ASSOCIATIVE PROPERTY

. a+(b+c)=(a+b)+c ∀ a,b,c €R

2 IDENTITY PROPERTY

For every a€R∃ 0€R ∋a+0 =a=0+a. Where 0 is identity element in R

3 INVERSE PROPERTY

For each a€R ∃–a€R ∋a+(-a)=0=(-a)+a. Where –a is inverse of a in R.

4 COMMUTATIVE PROPERTY

a+b=b+a∀a,b€R.

b) (R,•) is a semi group

. a•(b•c) =(a•b)• c ∀a,b,c€R

c) Distributive law's holds in R

1 a•(b+c) = a•b + a•c (LDL) ∀a,b,c €R

2 (b+c)•a = b•a + c•a (RDL)∀a,b,c€R

EXAMPLE:

1 (N,+,•) is not a Ring because the identity element '0' does not exists in N

2 (w,+,•) is not a Ring because the inverse does not exists in W

3 (Z,+,•) is a Ring

4 (Q,+,•) is a Ring

TYPES OF RINGS

RING WITH UNITY

Let (R,+,•) be a ring, if R satisfies for a€R ∃ 1€R ∍ a•1=a=1•a then R is said to be Ring with unity.

COMMUTATIVE RING

Let (R,+,•) be a ring, if R satisfies

a•b = b•a for a,b€R then R is called commutative Ring.

BOOLEAN RING

Let R be a Ring.If $a^2=a$ ∀ a€R then R is called BOOLEAN Ring.

THEOREM

Statement : Let R be a Ring 0,a,b€R then

I. 0a=a0=0

II. a(-b)=(-a)b=-ab

III. (-a)(-b)=ab

IV. a(b-c)=ab-ac

PROOF:

Let R be a Ring 0,a,b€R

CLAIM:

Case I $0a=a0=0$

Consider $0a=(0+0)a$

$0a=0a+0a$

$0=0a$

Similarly $0=a0$

$a0=0a=0$

Case ii $a(-b)=(-a)b=-ab$

Consider $a(-b)+ab$

$=a(-b+b)$

$=a0$

$=0$ (By case 1)

Now $a(-b)+ab=0$

$a(-b) = -(ab)$

Similarly $(-a)b= -(ab)$

Case iii $(-a)(-b)=ab$

Consider $(-a)(-b)$

$= -[a(-b)]$

$= -[-(ab)]$ (case ii)

$=ab$

$(-a)(-b) = ab$

Case iv. $a(b-c)=ab-ac$

Consider $a(b-c)$

$= a(b+(-c)$

$=ab+a(-c)$

$=ab-ac$

Theyfore $a(b-c)=ab-ac$