Understanding Subgroups

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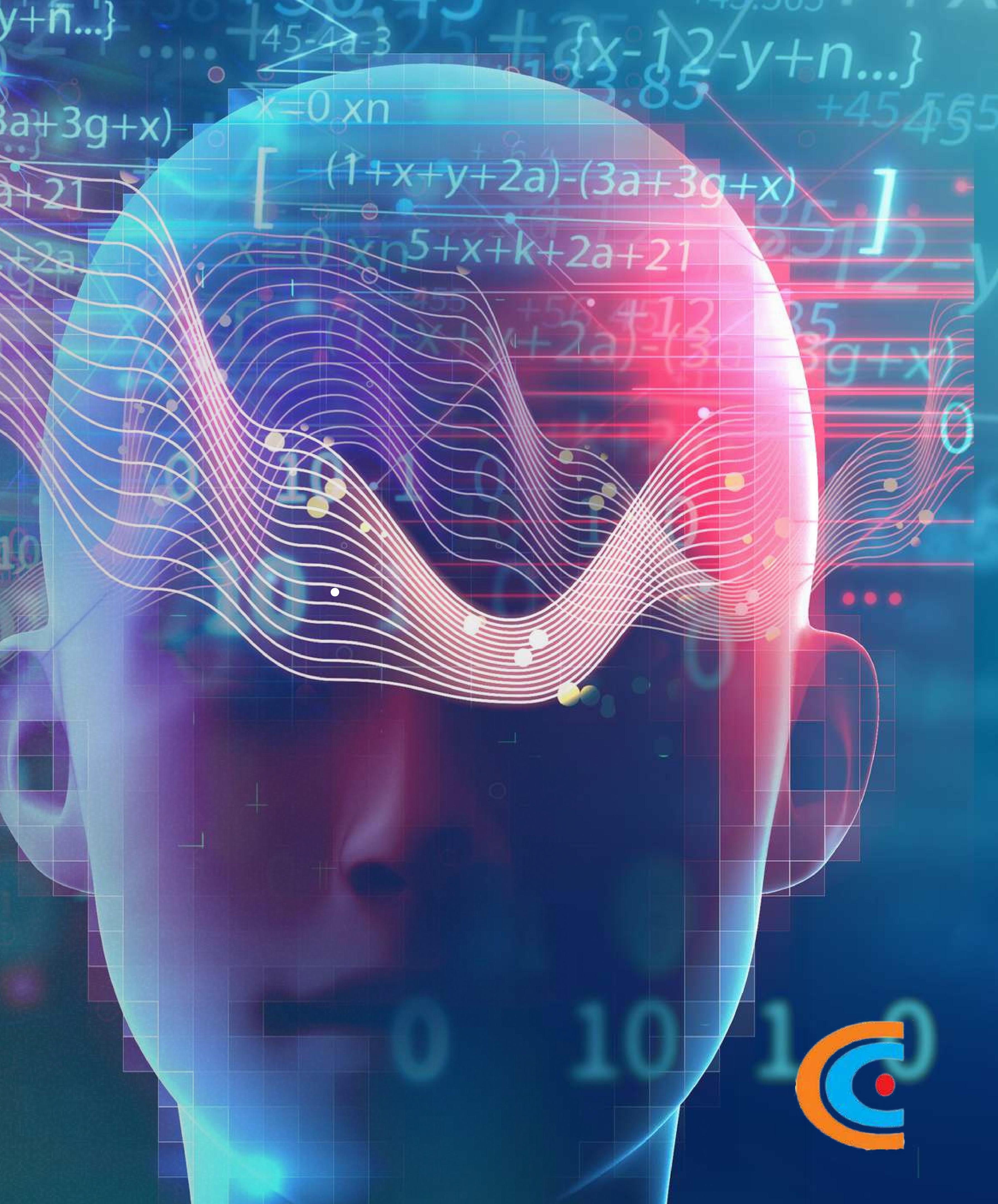
What are subgroups?

- Definition: A subgroup is a subset of a group that is itself a group under the same operation.
- Emphasize that a subgroup inherits the group structure from the original group.

Example of subgroup

- Present an example of a subgroup using a familiar group, such as the group of integers under addition.
- Show that the even integers form a subgroup of the integers.



Properties of subgroups

A subgroup is a subset of a larger group. It shares the same properties of the larger group, and must also satisfy closure, identity, and inverse axioms. Examples include the subgroup of even numbers in the group of integers, and the subgroup of symmetries in a geometric figure.

Subgroup criteria

Explain the criteria for a subset to be a subgroup:

- Closure: The subgroup must be closed under the group operation.
- Identity: The subgroup must contain the identity element of the group.
- Inverses: Every element in the subgroup must have its inverse in the subgroup.

Importance of subgroups

Present additional examples of subgroups from different groups:

- The cyclic subgroups in the group of integers or modular arithmetic.
- The rotational subgroups in the group of symmetries of a regular polygon.

Introduce the concept of cosets and Lagrange's Theorem.

- Explain that cosets partition the group into distinct sets, allowing for the study of group structure.
- State Lagrange's Theorem: In a finite group, the order of any subgroup divides the order of the whole group

Lagrange's Theorem

Statement: The order of the subgroup of a finite group divides order of a group.

Proof: let (G,.)be a group,(H,.)be a sub group of G

∴ G is finite H is also finite

If H=G

O(H)=O(G)

∴ O(H)/O(G)

If H ≠ G

Then O(H)   O(G)

letO(H)=m and O(G)=n

let H= $\{h_1,h_2,h_3..........h_m\}$ be the m distinct elements of H in G

Let a G

Ha=$\{h_1a,h_2a,h_3a........h_na\}$be the elements of Ha

If possible suppose that

$h_ia \neq h_ja$ for $i \neq j$

$h_i=h_j$

this is a contradiction for $h_i \neq h_j$ for $i \neq j$

our supposition is wrong

All the elements of Ha are distinct

∴ O(Ha)=m

Every right coset of H in G has exactly same m number of elements

since G is finite ,the distinct right cosets of H in G is also finite

let it be K

$Ha_1,Ha_2.........Ha_m$ are the distinct right cosets of H in G

$O(Ha_1)=O(Ha_2)=...........O(Ha_k)=m$

aa the right coset of H in G are disjoint

$G=Ha_1UHa_2U.........UHa_k$

$O(G)=O(Ha_1UHa_2U...........UHa_k)$

$O(G)=O(Ha_1)+O(Ha_2)+........+O(Ha_k)$

n=m+m+.........+m(ktimes)

n=km

O(H)/O(G)

∴ The order of a subgroup of a finite group divdes the order of a group

Theorem

If H and K are two subgroups of G then HK is also subgroup of G If HK=KH

Proof: let H and K are two subgroups of G

Suppose HK is a subgroup of G

Claim : HK=KH

Consider $HK=(HK)^{-1}$

$K^{-1}H^{-1}$

KH

Conversly suppose that HK=KH

Claim:HK is a subgroup of G

It is enough to prove that $(HK)(HK)^{-1}=HK$

(By known theorem ,H is a subgroupn of G Then $HH^{-1}=H$

Consider

$(HK)(HK)^{-1}=(HK)(K^{-1}H^{-1})$

$= H(KK^{1})H^{-1}$

$= HKH^{-1}$

$= (KH)H^{-1}$

$= K(HH^{-1})$

$= KH$

$HK(HK^{-1})=KH$

$(HK)(HK^{-1})=HK$

HK is a subgroup of G

Theorem

If it is any subgroup of G then $H=H^{-1}$
Proof :Let G be a group of G
Claim:$H=H^{-1}$
It is enough to prove that
HCH^{-1} and $H^{-1}CH$
Let $x \in H$
∴ H is a subgroup of G
H itself a group
by inverse $x^{-1} \in H$
$(x^{-1})^{-1} \in H^{-1}$
$x \in H^{-1}$
$H C H^{-1} \longrightarrow$ (1)
Let $y \in H^{-1}$
$y^{-1} \in H^{-1}$
$y^{-1} \in H$
By inverse $(y^{-1})^{-1} \in H$
$y \in H$
$H^{-1} C H \longrightarrow$ (2)
From 1 and 2
$H=H^{-1}$
Hence proved.

Conclusion

- Recap the key points discussed about subgroups.
- Reinforce the idea that subgroups provide insights into the structure and behavior of groups.
- Encourage further exploration of group theory and its applications.

Thank you

for considering the importance of understanding subgroups in our society. Let's continue to work towards creating a more inclusive and equitable world for all.