CHAITANYA
WOMEN'S COLLEGE
DEGREE & PG
AFFILIATED TO ANDHRA UNIVERSITY

TURING MACHINES

TMs model the computing capability of a general purpose computer, which informally can be described as:

- Effective procedure
  - Finitely describable
  - Well defined, discrete, "mechanical" steps
  - Always terminates
- Computable function
  - A function computable by an effective procedure

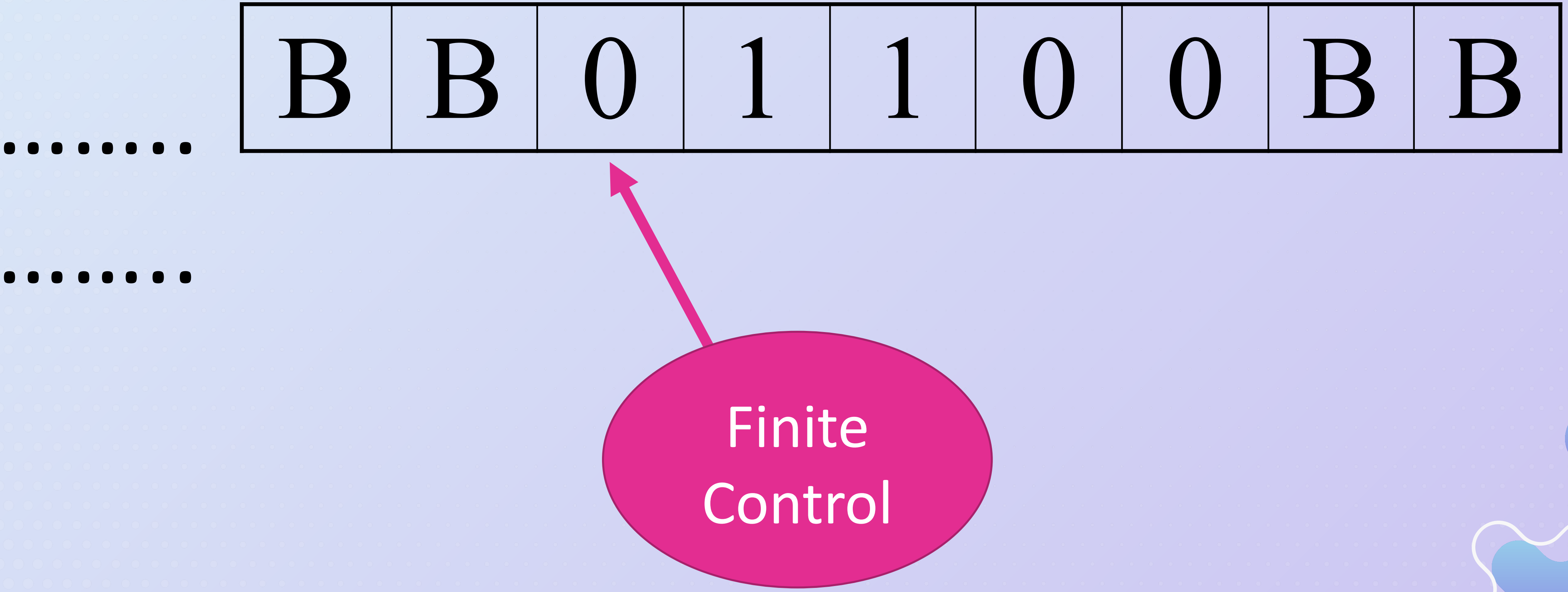
TMs formalize the above notion.

Church-Turing Thesis: There is an effective procedure for solving a problem if and only if there is a TM that halts for all inputs and solves the problem.

- There are many other computing models, but all are equivalent to or subsumed by TMs. *There is no more powerful machine* (Technically cannot be proved).

DFAs and PDAs do not model all effective procedures or computable functions, but only a subset.

Deterministic Turing Machine (DTM)



If read 1, write 0, go right, repeat.

If read 0, write 1, HALT!

If read •, write 1, HALT!

1	1	1	1	0	1				

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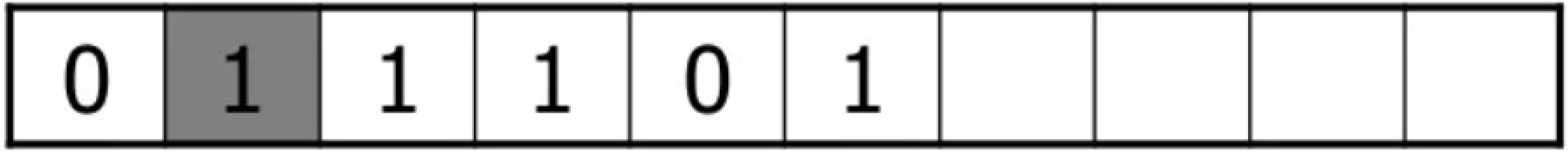

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So the successor's output on 111101 was 000011 which is the reverse binary representation of 48.

Similarly, the successor of 127 should be 128:

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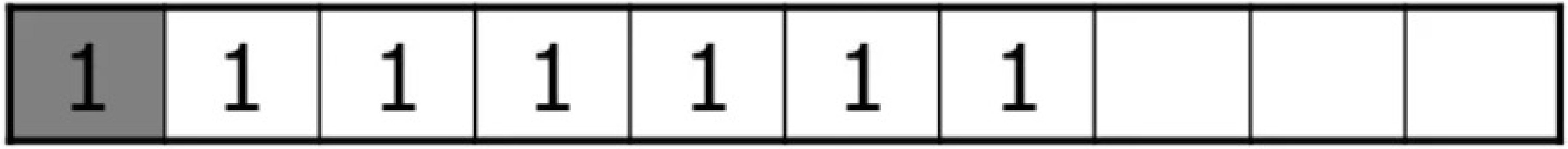

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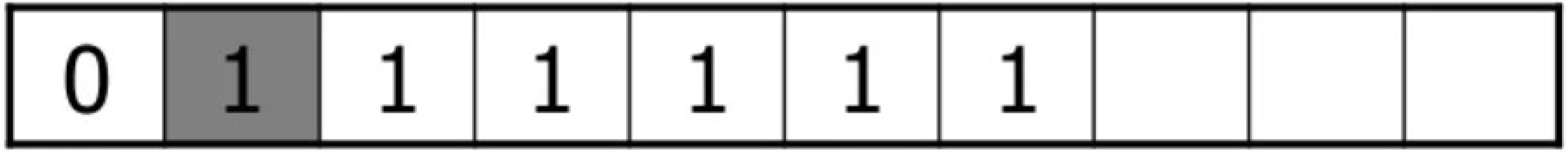

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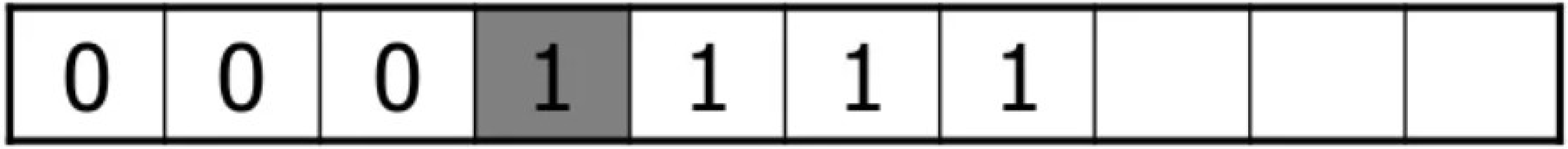

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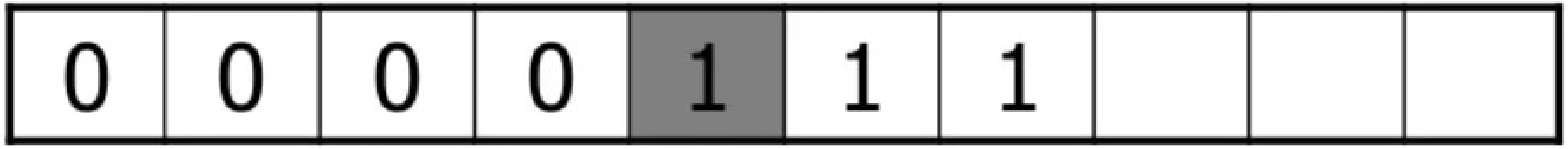

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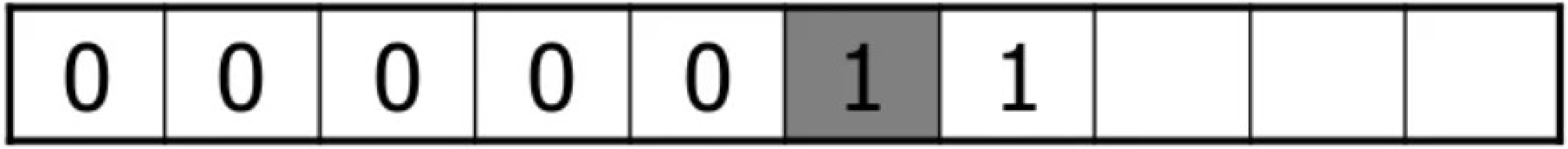

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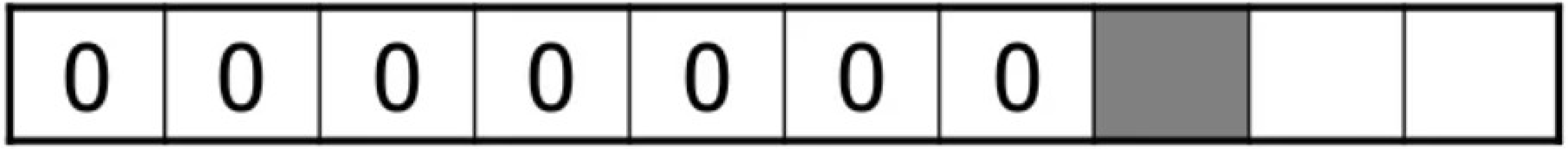

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- Two-way, infinite tape, broken into cells, each containing one symbol.
- Two-way, read/write tape head.
- An input string is placed on the tape, padded to the left and right infinitely with blanks, read/write head is positioned at the left end of input string.
- Finite control, i.e., a program, containing the position of the read head, current symbol being scanned, and the current state.
- In one move, depending on the current state and the current symbol being scanned, the TM

  1) changes state,

  2) prints a symbol over the cell being scanned, and

  3) moves its’ tape head one cell left or right.
- Many modifications possible, but Church-Turing declares equivalence of all.

Formal Definition of a DTM

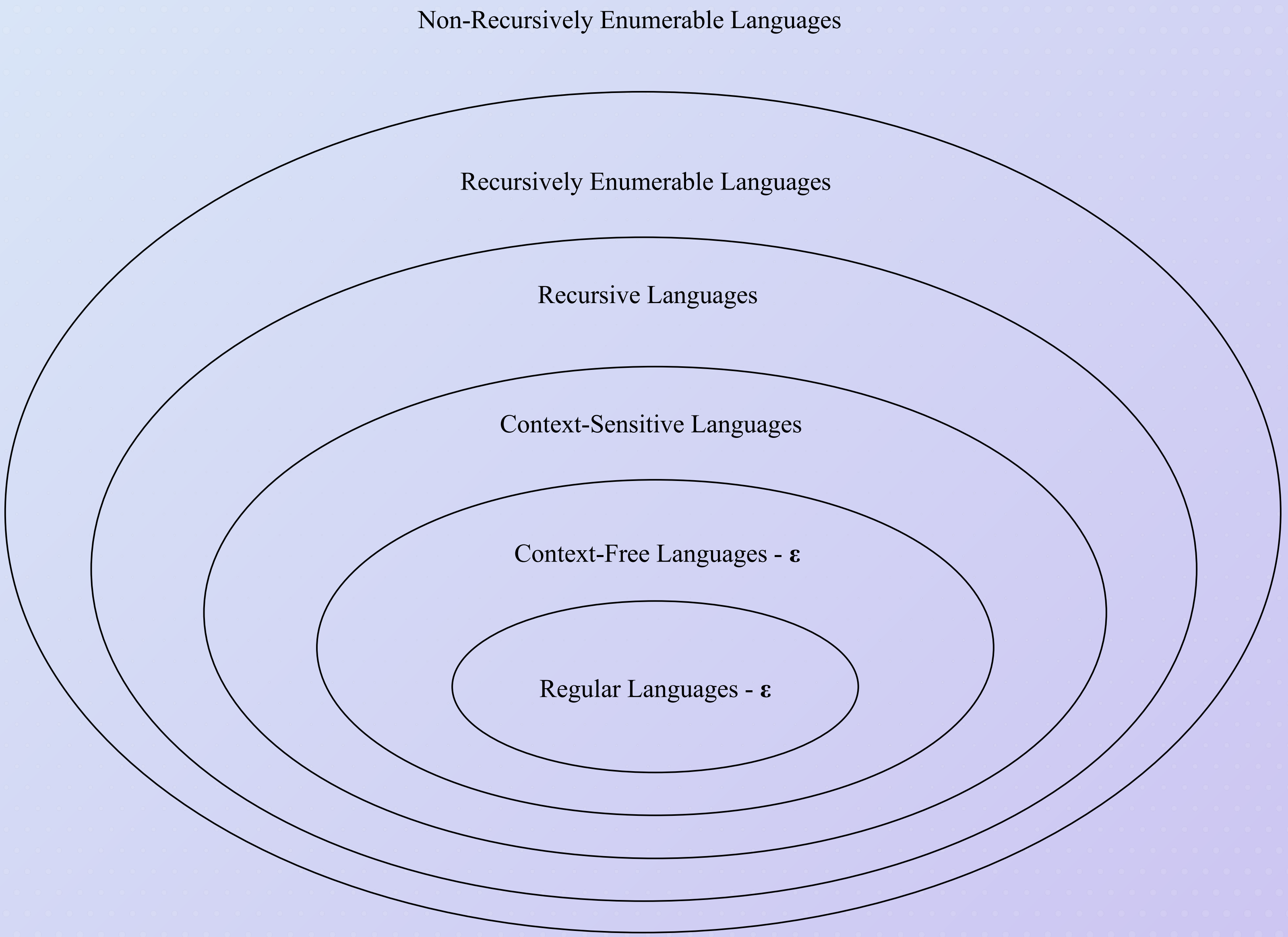
- A DTM is a seven-tuple:

  $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

Q	A finite set of states
Σ	A finite input alphabet, which is a subset of $\Gamma - \{B\}$
Γ	A finite tape alphabet, which is a strict superset of Σ
B	A distinguished blank symbol, which is in Γ
q_0	The initial/starting state, q_0 is in Q
F	A set of final/accepting states, which is a subset of Q
δ	A next-move function, which is a *mapping* (i.e., may be undefined) from

  $Q \times \Gamma \rightarrow Q \times \Gamma \times \{L,R\}$

  Intuitively, $\delta(q,s)$ specifies the next state, symbol to be written, and the direction of tape head movement by M after reading symbol s while in state q.


Non-Recursively Enumerable Languages
Recursively Enumerable Languages
Recursive Languages
Context-Sensitive Languages
Context-Free Languages - ε
Regular Languages - ε

The Halting Problem

- **Definition:** A decision problem is a problem having a yes/no answer (that one presumably wants to solve with a computer). Typically, there is a list of parameters on which the problem is based.
  - Given a list of numbers, is that list sorted?
  - Given a number x, is x even?
  - Given a C program, does that C program contain any syntax errors?
  - Given a TM (or C program), does that TM contain an infinite loop?

From a practical perspective, many decision problems do not seem all that interesting. However, from a theoretical perspective they are for the following two reasons:

- Decision problems are more convenient/easier to work with when proving complexity results.
- Non-decision *counter-parts* can always be created & are typically at least as difficult to solve.